Introduction to Logic:

Argumentation and Interpretation

Vysoká škola mezinárodních a veřejných vztahů

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Introduction to Logic: Argumentation and Interpretation

Annotation

The course offers an overview of topics in logic, communication, reasoning, interpretation and summary of their practical use in communication. It provides basic orientation in terminology of linguistic research and communication, persuasion and communication strategies, understanding the logic games, exercises and tasks, and offers the opportunity to learn the reasoning applied in various situations. The aim is that students not only get familiar with lectures, but also acquire the means of communication and argumentation through exercises and online tests.

Topics

1. Brief history of Logic and its place in science
2. Analysis of complex propositions using truth tables
3. The subject-predicate logic – Aristotelian square
4. Definitions and Terminology
5. Polysemy, synonymy, homonymy, antonymy
6. Analysis of faulty arguments
7. Interpretation – rules and approaches
8. Analysis of concrete dialogue

http://mediaanthropology.webnode.cz/kurzy/introduction-to-logic/

Predicate logic

In propositional logic it does not depend on the internal structure of the stament (proposition), because propositional logic deals only with those structures (complex statements, arguments), whose truthfulness or accuracy depends only on how they are connected to each other. Only a small part of the judgments can be formalized and proved in the context of propositional logic.

In predicate logic it depends on the internal structure of the verdict. If the correctness of the arguments depends on the internal structure of simple statements, the essential elements on which it depends, we call **terms**.

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Predicate logic

Aristotelian syllogisms consist of two simple premises and a simple conclusion.

Peter is a student.

Students are wise.

Peter is wise.

Every human is fallible.

John is human.

John is fallible.

If we mark these sentences as p, q, r, then the attempt to formalize within propositional logic is given by the following judgment: p, q / r, which corresponds to the formula: **(p ^ q) Þ r**

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Predicate logic – Aristotelian square

This formalization, however, is apparently insufficient for the following reasons:

The three statements are according to propositional logic elementary and independent of each other, but in fact they have internal components, they are structured, and between these components there is a connection. The term "man" is found in the statements p and q, the term "fallible" in the statements p and r, and the term "John" in the statements q and r.

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.
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Predicate logic – Aristotelian square

The formula **(p ^ q) Þ r** is not a tautology, the judgment, p, q / r is not valid, even if the judgment demonstrated by the example is valid. In predicate logic, which is a generalization of propositional logic, the judgment is formalized as

∀ x [p(x) Þ q(x)], p(J) |= q(J)

respectively by this formula: ∀ { x [p(x) Þ q(x)] **^** p(J)} Þ q(J)

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Predicate logic

- x is an object (individual) variable from a particular subject area - the universe of the discourse
- J is individual constant of the subject area (in the example a concrete person Jan)
- p, q are certain properties of objects from the universe of the discourse (in the example there are interpreted as characteristics of thinking beings „to be human" and „to be fallible"), p (x), q (x), respectively p (J), q (J) denotes that x respectively J has the property of p, respectively of q,
- The formula for **all the x []** indicates that for every individuals from the subject area it applies what is stated in brackets.

Sources: E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Predicate logic

- 1st order predicate logic formalizes judgments about the properties of objects and relationships between objects fixed by the subject area (universe).
- Predicate logics of the second and higher-order deal with the formalization of judgments, with properties of properties and with relations (and relations between properties and relations).
- 1st order predicate logic is a generalization of propositional logic, which can be considered a logic of the 0th order.

Source: E-vyuka pro logiku. [online] Online: http://snug.ic.cz/index.htm#

Predicate logic

There are two kinds of terms:

- **General** – linguistic expressions that indicate a larger number of subjects, ie. a set of objects (man, city, number, ...).
- **Singular** – linguistic expressions that denote just one object (names such as Paul, Prague ...).

Singular Statements

A simple singular statement is formed by the merger (connection) of a singular term with a general term with the connective = "is".

Example:

Paul is a human. (ie. Paul belongs to the group of people)

Prague is a city. (ie. Prague belongs to the set of cities)

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic

Singular simple statement is a simple method of prediction, whose purpose is to testify something (man, city,...) about something (Paul, Prague, ...)   (predicate, subject)

On simple singular statements the rules of propositional logic can be applied, ie. we can combine them into complex propositions using logical connectives.

A general term "human" can be associated (connected) with only one singular term (one variable):

Peter is human.

It is therefore a **single predicate**.

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic

There are also general terms that relate to arranged couples.

They do not express their properties, but their relationship ("greater than", "brother of", ...)

Peter is greater than Paul.

(singular term) (genal term) (singular term)

Ivan is the brother of Ondra.

Such general terms are called **relational terms**.

There exist also relational terms that express the relation between more than two subjects. To set up such a simple statement we need the appropriate number of singular terms. It is an n-digit predicate. To express the relationship that Prague lies between Melnik and Benesov we need a minimum of three singular terms Prague, Melnik, Benesov.

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic

Alphabet of predicate logic consists of the following groups of symbols:

a) Logical symbols

- subject (individual) variables: x, y, z,... (resp. with index)
- symbols for connectives: ¬ ⌐ ^ ˅ Þ ≡
- symbols for quantifiers: ∀ for all, ∃ exists
- event. binary predicate symbol = (predicate logic with equality)

b) Special symbols (determine the specifics of the language)

- predicate symbols: p, q, r,... (resp. with index)
- functional symbols: f, g, h,... (resp. with index)

Source: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

Predicate logic

For each functional and predicate symbol there is assigned a non-negative number *n* ($n \geq 0$), so called **arita**, giving us the number of individual variables that are arguments to the function or predicate.

c) Auxiliary symbols /brackets/: (,) / eventually. [,],{,}/

Souce: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

Predicate logic – Grammar

a) terms: each symbol of variable is a term

where *t1, ..., tn* (n ≥ 0) are terms and where *f* is *n-ary* function symbol, then the expression *f (t1, ..., tn)* is a term; for n = 0 is a null function symbol or individual constant (marked a, b, c, ...)

b) an atomic formula:

if *p* is *n-ary* predicate symbol and if *t1, ..., tn* are terms, then the expression *p (t1, ..., tn)* is an atomic formula

where *t1* and *t2* are terms, then the expression *(t1 = t2)* is an atomic formula

Source: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

Predicate logic – Grammar

c) formula:

each atomic formula is a formula:

- if the expression *A* is a formula, then ¬ *A* is a formula
- if the expressions *A* and *B* are formulas, then expressions *(A disjunction B)*, *(A Conjunction B)*, *(A implication B)*, *(A ≡ B)* are formulas
- if *x* is a variable and *A* formula, then expressions *for all xA* and *exists xA* are formulas

Source: *E-vyuka pro logiku.* [online] Online: http://snug.ic.cz/index.htm#

Predicate logic – Grammar

In predicate logic parts form the structure by predicate constant (F) and individual constant (a). Truthfulness depends here on whether we combine suitable predicate constant with suitable individual constant.

What is the appropriate connection?

The statement in the form F (a) is true if and only if there is a subject indicated by individual constant that is element of the set indicated by predicate constant.

Statement "*Zinc is a chemical element*" is true if Zinc belongs to a set of chemical elements. Statement is false, if Zinc does not belong to a set of chemical elements.

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic – Grammar

The truth value of the statement F (a) can be explained also by Frege's approach, through the predicate function.

E.g. "*It is a chemical element*" we perceive as a function that when applied for Zinc, it is true (Zinc is a chemical element). If it is applied eg. for the water, it is false (*water is a chemical element* is a false statement).

Approaches

- using a function that is assigning values *truth*, *false*
- using a set and properties *belonging to it*, *not to belong to it*

upraveno podle: : **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic – General statements

In natural language we find statements of a general nature, for example: Everyone is mortal, some dogs are dachshunds, no man is an animal ...

Analysis of the statement *Everyone is mortal.*

F ... *be mortal* (predicate constant)

Individual constant – not obvious,

it is only undefined term *each*

How do we use this term?

If we examine some set of individuals, eg. a set of people, the statement *everyone is mortal* is true if and only if every element of the setm is mortal.

Sources: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.
https://en.wikipedia.org/wiki/Dachshund

Predicate logic – General statements

Peter is mortal.

Ivan is mortal.

Jane is mortal.

etc. for all such elements of the set of people, we decided to investigate.

Verdict: For every x: x is mortal.

F (x) ... x is mortal

For every x: F (x)

$\forall$ symbol for the expression *for each* (universal quantifier)

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic – PARTIAL STATEMENTS

Statements that begin eg. with the expression *some*.

Some dogs are dachshunds.

Two predicate constants G ... dog, F .... dachshund

We are looking for individuals who have the property (characteristics) that simultaneously *x is a dog* and *x is a dachshund*.

For the statement to be true, it is sufficient that there is at least one such individual who is both a dog and the dachshund.

There is at least one x for which: x is a dog and x is a dachshund.

There is at least one x, for which: G (x) ^ F (x)

∃ symbol for the expression *there is at least one* (existential quantifier) **∃ x (G (x) ^ F (x))**

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic

When using quantifiers we have to consider every individual of the field of our consideration (universe of discourse).

Criteria of the field of consideration:

1. It must contain items that are marked by individual constants.
2. It must contain items that are marked by predicate constants.
3. The field of consideration must not be empty.

Examples:

Every man is mortal. (field of consideration: set of animals)

Some dogs are dachshunds. (field of consideration: animals)

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic

By predicate logic we can express for example Aristotelian statements.

We examine, for example, a set of animals (field of consideration):

Each dog is a mammal.

No dog is a mammal.

Some dog are mammals.

Some dog are not mammals.

Two general terms

F...... dog

G .... mammal

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic

	Natural language	Aristotelian	Predicate logic
(1)	Each dog is a mammal.	F a G	∀ x (F(x) Þ G(x))
(2)	No dog is a mammal.	F e G	∀ x (F(x) Þ ¬ G(x))
(3)	Some dog are mammals.	F i G	∃ x (F(x) ^ G(x))
(4)	Some dog are not mammals.	F o G	∃ x (F(x) ^ ¬ G(x))

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic

Field of consideration = a set of three elements (a,b,c) Individuals: a,b,c

Property F (predicate constant)

Statement $\forall x\ F(x)$

$\forall x\ F(x) \equiv (F(a) \wedge F(b) \wedge F(c))$

universal quantifier expressed using **conjunction**

Statement $\exists x\ F(x)$

$\exists x\ F(x) \equiv (F(a) \vee F(b) \vee F(c))$

existential quantifier expressed using **disjunction**

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic – Rules

The rule of general quantifier elimination:

- If we have a formula in the form: ∀ x F (x), we can move on to Formula F (a) (in the sense of truthfulness it indicates a particular object)

If every human is an animal, then Peter is an animal. (Peter is from the set of humans.)

The rule of introduction of existential quantifier:

- If we have a formula in the form F (a), we can move on to the formula ∃ x F (x)

(Applies only when it denotes a particular object.)

If Peter is an animal, then a human is an animal. (Peter is human.)

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic – Rules

The rule of the relationship between the general and existential quantifier:

If we have a formula in the form: $\forall$ x F (x), we can move on to the formula $\exists$ x F (x)

(Applies when the field of consideration is not empty)

If every human is an animal, then some human are animals.

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic – Rules

The relation between conjunction and disjunction (de Morgan's Law)

$(F(a) \wedge F(b) \wedge F(c)) \neg (\neg F(a) \vee \neg F(b) \vee \neg F(c))$

$\forall x \; F(x) \equiv \neg \; (\exists x \; \neg F(x))$

$\neg \forall x \; F(x) \equiv \; x \exists \neg \; F(x)$

$\forall x \neg \; F(x) \equiv \neg (\exists x F(x))$

$\neg (\forall x \neg F(x)) \equiv \exists x \; F(x)$

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Predicate logic – Rules

- Formula with existential quantifier is associated with the disjunction
- Formula with a general quantifier is associated with conjunctions

the relation between implication and conjunction:

$(p \Rightarrow \forall q) \equiv \neg (p \wedge \neg q)$

$(p \wedge q) \equiv \neg (p \Rightarrow \neg q)$

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

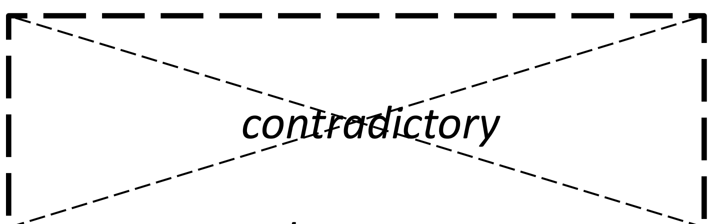
Predicate logic – Aristotelian statements in predicate logic

Each dog is a mammal. $\forall x\ (F(x) \Rightarrow G(x))$ $\neg \exists x\ (F(x) \wedge \neg G(x))$	No dog is a mammal. $\forall x\ (F(x) \Rightarrow \neg G(x))$ $\neg \exists x\ (F(x) \wedge G(x))$
Some dog is a mammal. $\exists x\ (F(x) \wedge G(x))$ $\neg \forall x\ (F(x) \Rightarrow \neg G(x))$	Some dog is not a mammal. $\exists x\ (F(x) \wedge \neg G(x))$ $\neg \forall x\ (F(x) \Rightarrow G(x))$

aff**i**rmo (claim, *lat.*) – n**e**gg**o** (deny, *lat.*), **subject (S)** a **predicate (P)**

Each (+) *contrary* **None** (-)

S a P **S e P**



subalternate *contradictory* *subalternate*

Some (+) *subcontrary* **Some** (-)

S i P **S o P**

Source: **NYTROVÁ, Olga - PIKÁLKOVÁ, Marcela.** *Etika a logika v komunikaci.* Praha: UJAK, 2007.

Tasks

Which terms can be described as singular?

a) Vltava, river, Barikádníků Bridge, Charles University

b) love, happiness, freedom, peace, God

c) Bratří Synků Square, Vltava, Jan Amos Comenius

d) Charles University, J. A. Comenius Square, teacher of nations

e) neither of these

Tasks

Draw the statement into the sets and decide which statement results from the premises *Charles does not like any romantic movies.* and *Charles does not like the film Braveheart.*

a) Charles likes some romantic movies.

b) Charles does not watch any movies.

c) Charles likes some movies.

d) Romantic movies are boring.

e) None of the options.

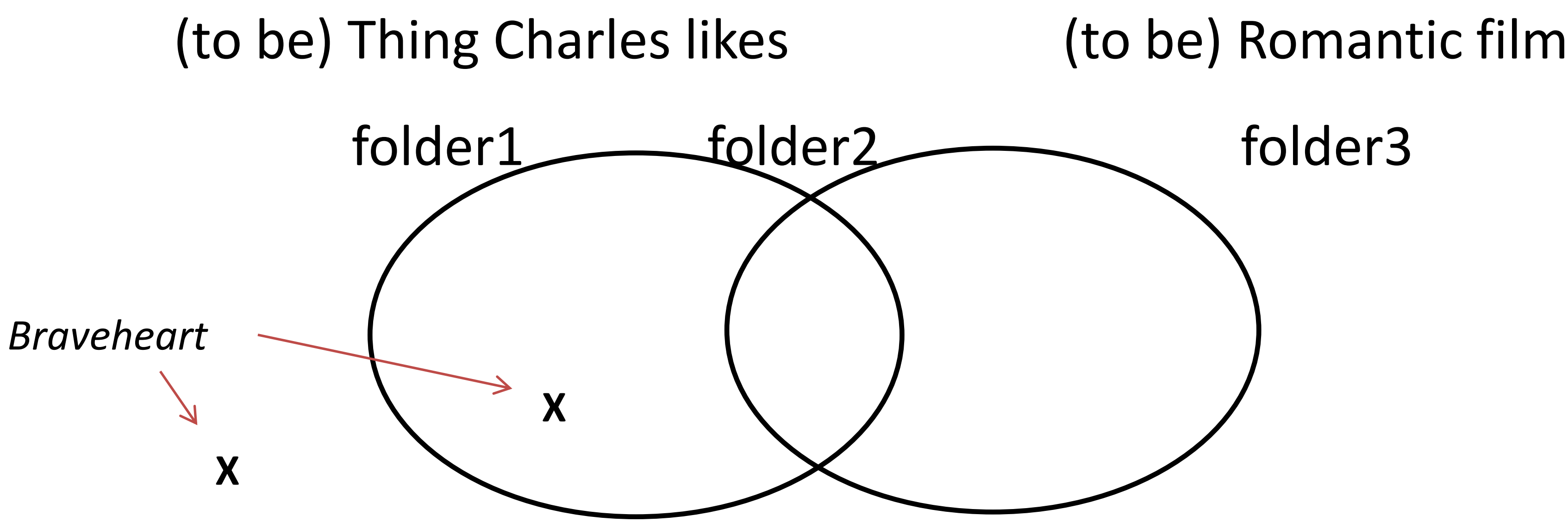
Introduction to Logic: Argumentation and Interpretation

Tasks

Charles does not like any romantic movies

Charles does not like the film Braveheart.

Number of sets = number of given properties



if there is a general and partial statement, we must begin with the general

(folder2 is empty)

Tasks

Draw the statement into the sets and decide which statement results from the premises *Charles does not like some romantic movies.* and *Charles does not like horror movies.*

a) Charles likes some romantic movies.

b) Charles does not like the movie Twilight.

c) Peter also does not like romantic movies.

d) Romantic movies are boring.

e) None of the options.

Thank you for your attention!

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In case of a need, don´t hesitate to contact me:

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